55.

$y' = e^{\frac{x-y}{2}}$

$l_1 \perp l_2 \iff m_1 m_2 = -1$

$m_1 = \frac{-1}{m_2}$

$y' = -e^{\frac{y-x}{2}}$

$\left(e^{\frac{x-y}{2}}\right)\left(-e^{\frac{y-x}{2}}\right)$

$\frac{-1}{e^{\frac{x-y}{2}}} = -1 \cdot e^{\frac{-(x-y)}{2}}$

$= -e^{\frac{-x+y}{2}}$

$-e^{\frac{x-y+y-x}{2}} = -e^0$

$= -1$

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65 $y' = x - \frac{1}{x^2}$ $y(1) = 2$

a) $y' = x - x^{-2}$

$y = \frac{x^2}{2} - \frac{x^{-1}}{-1} + c$

$y = \frac{x^2}{2} + \frac{1}{x} + c$

$2 = \frac{1}{2} + 1 + c$

$\frac{1}{2} = c$

b) $y(-1) = 1$

$1 = \frac{1}{2} - 1 + c$

$\frac{3}{2} = c$

c) $y = \begin{cases} \frac{x^2}{2} + \frac{1}{x} + C_1, & x < 0 \\ \frac{x^2}{2} + \frac{1}{x} + C_2, & x > 0 \end{cases}$

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6.2 antiderivatives by substitutions

memorize the basic formulas p 332

$$\int u^n \, du = \frac{u^{n+1}}{n+1} + c$$

what if u is a function of x

$$\int (\sin x)^3 \cos x \, dx = \int u^3 \, du = \frac{u^4}{4} + c$$

$u = \sin x$ $du = \cos x \cdot dx$

$$= \frac{(\sin x)^4}{4} + c$$

$$\int (\sin x)^3 \cos x \, dx = \frac{(\sin x)^4}{4} + c$$

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$$\int e^{\tan x} \sec^2 x \, dx$$

let $u = \tan x$

$du = \sec^2 x \, dx$

$$\int e^u \, du = e^u + c$$

$$= e^{\tan x} + c$$

$y = e^{\tan x}$

$y' = \sec^2 x \cdot e^{\tan x}$

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$$\int \cot x \, dx \qquad \int \frac{1}{u} du$$

$$\int \frac{\cos x}{\sin x} dx = \int \frac{du}{u} = \ln|u| + c$$

$u = \sin x$

$du = \cos x \, dx$

$$\int u^{-1} du$$

final answer $= \ln|\sin x| + c$

$$dy = \frac{dy}{dx} dx$$

check: derivative $= \frac{1}{\sin x} \cdot \cos x$

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$$\int \cot(7x) \, dx$$

$$\frac{1}{7} \int 7 \frac{\cos(7x)}{\sin(7x)} dx = \frac{1}{7} \int \frac{du}{u} = \frac{1}{7} \ln|u| + c$$

let $u = \sin(7x)$

$du = 7\cos(7x) \, dx$

$$= \frac{1}{7} \ln|\sin(7x)| + c$$

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$$\frac{1}{2}\int 2x\sqrt{5+x^2}\,dx = \frac{1}{2}\int \sqrt{u}\,du$$

$u = 5+x^2$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

$$= \frac{1}{2}\int u^{\frac{1}{2}}\,du$$

$$= \frac{1}{2}\,\frac{u^{3/2}}{3/2} + c$$

$$= \frac{1}{2}\cdot\frac{2}{3}\left(5+x^2\right)^{3/2} + c$$

$$= \frac{\left(5+x^2\right)^{3/2}}{3} + c$$

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$$\frac{1}{2}\int_0^1 \frac{2x}{x^2-4}\,dx = \frac{1}{2}\int_{-4}^{-3}\frac{du}{u} = \frac{1}{2}\ln|u|\Big|_{-4}^{-3}$$

✓ $u = x^2-4$

$\frac{du}{dx} = 2x$

$du = 2x \cdot dx$

$x=0 \quad u=-4$

$x=1 \quad u=-3$

$$= \frac{1}{2}\ln|-3| - \frac{1}{2}\ln|-4|$$

$$= \frac{1}{2}\left(\ln 3 - \ln 4\right)$$

$$= \frac{1}{2}\ln\frac{3}{4}$$

$$= \ln\left(\frac{3}{4}\right)^{1/2} = \ln\sqrt{\frac{3}{4}}$$

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